Jet and Rocket Propulsion

AE4451

LECTURE 5

Sedina Tsikata – January 20, 2023

Overview

what we saw last time:

- example applying the Reynolds Transport Theorem
  - turbine system

- energy state equations

today:

- thermodynamics continued: ideal gas mixtures and examples

Mixtures of perfect gases

Consider the perfect gas law, written as

$$pV = n\overline{R}T$$

$\overline{R}$ = universal gas constant

n = total number of moles

Gibbs-Dalton law

the pressure of the gas mixture of *n* constituents is the sum of the partial pressures of each constituent

i.e.

$$p = \sum_{i=1}^{n} p_i \qquad n = \sum_{i=1}^{n} n_i \qquad \chi_i = \frac{n_i}{n} = \frac{p_i}{p}$$

partial pressure

mole fraction

- we assume that each constituent is an ideal gas

Mixtures of perfect gases

This law allows us to write expressions for the energy, enthalpy and entropy, where

$$E = \sum_{i=1}^{n} E_i = \sum_{i=1}^{n} m_i e_i$$

energy

$$H = \sum_{i=1}^{n} H_i = \sum_{i=1}^{n} m_i h_i$$

enthalpy

$$S = \sum_{i=1}^{n} S_i = \sum_{i=1}^{n} m_i s_i$$

entropy

- the specific heats are then written

$$c_p = \frac{\sum_{i=1}^{n} m_i c_{pi}}{m}$$

$$c_v = \frac{\sum_{i=1}^{n} m_i c_{vi}}{m}$$

m_i = mass of constituent i

m = mass of mixture

Mixtures of perfect gases

- alternatively, we can also consider using <u>averaged</u> properties of the mixture, where:

universal

$$p = \rho R_{mix} T \qquad R_{mix} = \frac{\bar{R}}{MW_{mix}} \qquad MW_{mix} = \sum_i \chi_i MW_i \qquad \chi_i = \frac{n_i}{n_{mix}}$$

molar weight

Molar weight constituent

mole fraction constituent

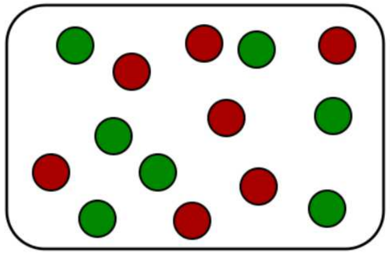
new gas constant for the mixture

MW = molar weight of mixture

mole fraction

Mixtures of perfect gases

- let's consider what these two treatments give us for enthalpy *H* for a particular mixture



mixture of two gases C

constituent A

constituent B

what is the enthalpy change from state 1 to state 2?

(s1) (s2)

C = mixture going from s1 to s2

1. summation over constituents

$$\Delta H_{C_{1,2}} = \Delta H_{A_{1,2}} + \Delta H_{B_{1,2}}$$

$$\Delta H_{C_{1,2}} = m_A\left[h_A(T_2) - h_A(T_1)\right] + m_B\left[h_B(T_2) - h_B(T_1)\right]$$

$$\Delta H_{C_{1,2}} = m_A \int_{T_1}^{T_2} c_{p_A}\, dT + m_B \int_{T_1}^{T_2} c_{p_B}\, dT$$

Mixtures of perfect gases

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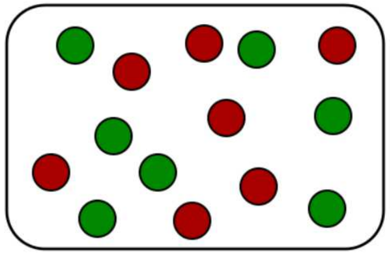
- let's consider what these two treatments give us for enthalpy *H* for a particular mixture



mixture of two gases C

constituent A

constituent B

what is the enthalpy change from state 1 to state 2?

<u>2. averaged properties of mixture</u>

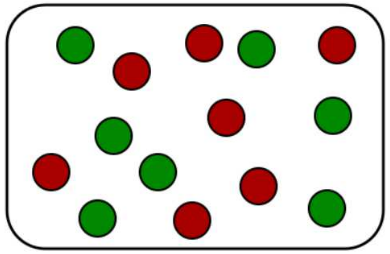
$$\Delta H_{C_{1,2}} = m_{mix} \int_{T_1}^{T_2} c_{p_{mix}} dT$$

$$c_{p\,mix} = \sum_i Y_i c_{p\,i}$$

where $Y_i = \dfrac{m_i}{m_{mix}}$ mass fraction

Mixtures of perfect gases

- now considering entropy S for this mixture



mixture of two gases C
constituent A
constituent B

from last class, Gibbs equation:

entropy

$$\left(\int_{s_1}^{s_2} ds = s_2 - s_1 = \int_{T_1}^{T_2} \frac{c_p(T)dT}{T} - R\int_{p_1}^{p_2} \frac{dp}{p}\right)$$

$$s_1 - s_2 = c_p \ln\frac{T_2}{T_1} - R\ln\frac{p_2}{p_1}$$

if calorically perfect gas

1. summation over constituents

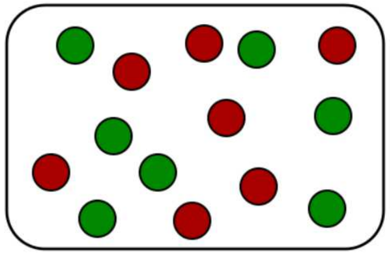
$$* \quad \Delta S_{C_{1,2}} = m_A\left\{\int_{T_1}^{T_2} c_{p_A}\frac{dT}{T} - R_A\ln\left(\frac{p_{2A}}{p_{1A}}\right)\right\} + m_B\left\{\int_{T_1}^{T_2} c_{p_B}\frac{dT}{T} - R_B\ln\left(\frac{p_{2B}}{p_{1B}}\right)\right\}$$

entropy

constituents are at the same temperature T
each constituent represented by its partial pressure *p*

Mixtures of perfect gases

- entropy *S* for this mixture if we consider averaged properties



mixture of two gases C

constituent A

constituent B

2. averaged properties of mixture

$$\Delta S_{C_{1,2}} = m_{mix}\left[\int_{T_1}^{T_2} c_{p_{mix}}\frac{dT}{T} - R_{mix}\ln\left(\frac{p_2}{p_1}\right)\right]$$

mass mixture

Examples

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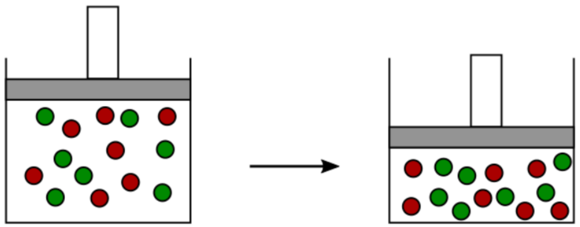


W_{in}

Q. If air at 1 atm and 300 K is compressed to 10~~0~~ atm and 750 K, find the change in entropy per unit mass

assumptions:

- synthetic air is 79% N_2 and 21% O_2 (by mole)
- N_2, O_2 are calorically perfect gases under these conditions

1. summation over constituents

$$\Delta S_{12,air} = \Delta S_{12,N_2} + \Delta S_{12,O_2}$$

$$m_{air}\Delta s_{12,air} = m_{N_2}\Delta s_{12,N_2} + m_{O_2}\Delta s_{12,O_2}$$

$$\Delta s_{12,air} = Y_{N_2}\Delta s_{12,N_2} + Y_{O_2}\Delta s_{12,O_2}$$

mass fraction

N_2 = diatomic

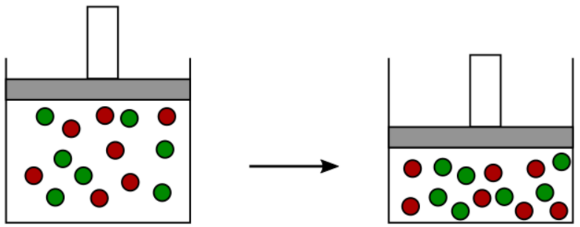
14 + 14

$$Y_{N_2} = \chi_{N_2}\frac{MW_{N_2}}{MW_{air}} = 0.79\frac{28.01}{28.8} = 0.767$$

$O_2 = (1 - 0.767)$

notice we go from mole to mass fraction here

Examples



Q. If air at 1 atm and 300 K is compressed to 10 atm and 750 K, find the change in entropy per unit mass

assumptions:

- synthetic air is 79% N_2 and 21% O_2 (by mole)
- N_2, O_2 are calorically perfect gases under these conditions

$R = \frac{J}{(kmol\,K)}$

1. summation over constituents

$$\Delta s_{12,N_2} = \frac{8314}{28.01}\frac{J}{kg_{N_2}K}\left\{\frac{7}{2}\ln\left(\frac{750}{300}\right) - \ln\left(\frac{7.9}{0.79}\right)\right\}$$

$$= 0.268\, kJ/kg_{N_2}K$$

$$\Delta s_{12,O_2} = \frac{8314}{32.0}\frac{J}{kg_{O_2}K}\left\{\frac{7}{2}\ln\left(\frac{750}{300}\right) - \ln\left(\frac{2.1}{0.21}\right)\right\}$$

$$= 0.235\, kJ/kg_{O_2}K$$

mass fract.

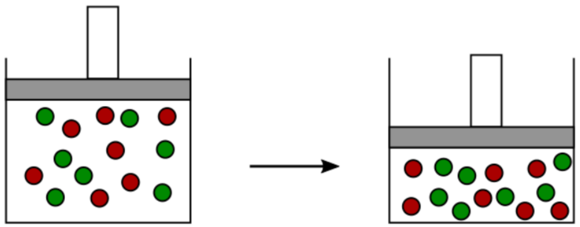
recall diatomic $c_p = \frac{7R}{2}$

$$\Delta s_{12,air} = Y_{N_2}\Delta s_{12,N_2} + Y_{O_2}\Delta s_{12,O_2}$$

$$= 0.767\frac{kg_{N_2}}{kg_{air}}\left(0.268\frac{kJ}{kg_{N_2}K}\right) + 0.233\frac{kg_{O_2}}{kg_{air}}\left(0.235\frac{kJ}{kg_{O_2}K}\right)$$

$$\boxed{\Delta s_{12,air} = 0.26\frac{kJ}{kgK}}$$

Examples



Q. If air at 1 atm and 300 K is compressed to 10 atm and 750 K, find the change in entropy per unit mass

assumptions:

- synthetic air is 79% N_2 and 21% O_2 (by mole)
- N_2, O_2 are calorically perfect gases under these conditions

2. using averaged properties

28.01

$$MW_{air} = 0.79(28\,kg/kmol) + 0.21(32\,kg/kmol) = 28.85\,kg/kmol$$

$$R_{air} = \frac{\overline{R}}{MW_{air}} = \frac{8314\,J/kmolK}{28.8\,kg/kmol} = 288\,J/kgK$$

$$c_{p_{air}} \cong \frac{7}{2} R_{air} = 1.01\,kJ/kgK$$

$$\Delta s_{12} = c_{p_{air}} \ln\left(\frac{T_2}{T_1}\right) - R_{air} \ln\left(\frac{p_2}{p_1}\right)$$

$$= 1.01 \frac{kJ}{kgK} \ln\left(\frac{750}{300}\right) - 0.288 \frac{kJ}{kgK} \ln\left(\frac{10}{1}\right)$$

$$\boxed{\Delta s_{12,air} = 0.26 \frac{kJ}{kgK}}$$